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BENDING MOMENT ANALYSIS OF RECTANGULAR THIN PLATE USING IMPROVED FINITE DIFFERENCE METHOD

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Abstract: **This research analysed the bending moment of rectangular thin plate having all edges simply supported. The analysis was accomplished through the development of the mathematical model for bending moment acting along X and Y-axis of a rectangular thin plate using the improved finite difference approach by the method of higher approximation. The improved finite difference model for bending moment was developed using Taylor's series expression. The results obtained using the model for X and Y-axis was compared with the ordinary finite difference solution and the exact solution to check the accuracy of the newly developed model. The results emerging from the improved finite difference model for the X-axis showed an average percentage error of -0.048% to the exact solution while the result emerging from ordinary finite difference model showed -3.035% from the exact solution. The result of the improved finite difference for the Y-axis showed an average percentage error of -0.798% while the ordinary finite difference showed -0.912% to the exact solution. Hence the results suggest that the improved finite difference model approximated closely to the exact method than the ordinary finite difference; thus the improved finite difference model proved to be more accurate than the existing ordinary finite difference model. Therefore the improved finite difference model developed for bending moment is effective and is recommended for use in structural engineering.**

Keywords: **Improved Finite Difference, Taylor Series, Bending moment, Rectangular thin plate, Mathematical model, Convergence, Exact solution.**

LIST OF SYMBOLS:

'D' is the flexural rigidity, $'p_z'$ is the applied transverse load,$'v'$ is the passion ratio, $'E'$ is the modulus of elasticity and $'h'$ is the plate thickness, $\Delta x =$ distance along the x-axis, $\alpha =$ aspect ratio, $\Delta y =$ distance along y-axis, $m =$ length per each mesh and $n =$ width per each mesh.

1. INTRODUCTION

Analysis or solution to a plate problem means the determination of stress at various points of the plate. Thus, plate analysis ensures the suitability of the plate to resist the design load (Roknuzzaman et al, 2015). Plate bending refers to the deflection of a plate perpendicular to the plane of the plate under the action of external forces and moments. The amount of bending can be determined by solving the differential equations of the plate theory (Baglekar and Deshmukh, 2014).

A number of research works performed on the solution of plate problems based on analytical and numerical methods in the past years described the analytical method as the only method that gives the exact value of the plate behaviour while the numerical method gives an approximate or close value (Ezeh et al, 2013). Szilard (2004) used ordinary finite difference method to get the solution for bending moment analysis of thin rectangular flat plate having various boundary conditions. From the research works, it was indicated that the convergence characteristics of the ordinary finite difference method converge slowly towards the exact solution of a given plate problem. Hence he (Szilard) suggested when higher accuracy in the finite difference solution of plate problems is required; the improved finite difference technique is recommended (Szilard, 2004).

In this research work, a rectangular thin plate with all edges simply supported (SSSS) was analysed using the improved finite difference method and the results were compared with the ordinary finite difference solution and the exact solution.

2. LITERATURE REVIEW

2.1 Thin Plate Theory:

The governing differential equation for bending moment acting along x and y axis for rectangular thin plates was given by Szilard (2004) as

$$M_x = -\frac{Eh^3}{12(1-v^2)}\left(\frac{d^2w}{dx^2} + v\frac{d^2w}{dy^2}\right) = P_z \quad (2.1)$$

$$M_y = -\frac{Eh^3}{12(1-v^2)}\left(\frac{d^2w}{dy^2} + v\frac{d^2w}{dx^2}\right) = P_z \quad (2.2)$$

where

$$\frac{Eh^3}{12(1-v^2)} = D \quad (2.3)$$

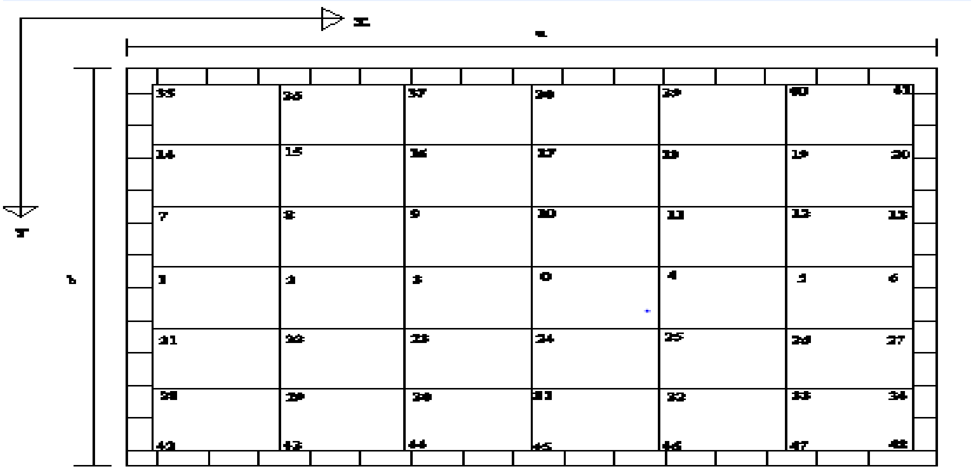
2.2 Taylor Series:

The Taylor series expansion are expressed as

$$T(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots \quad (2.4)$$

(Szilard, 2004; Math-image, 2013; Ibearugbulem et al, 2014; Umeonyiagu et al, 2017).

3. MATERIALS AND METHOD



$v = 0.3$

Figure 3.1: Diagram showing equally spaced Discretized Rectangular Plate

3.1 Formulating the Expression for the Improve Finite Difference along the X-axis and Y-axes

The derived governing differential equation for bending moment shown in equation 2.1 and 2.2 is of second order derivatives acting along x and y axis, hence the improved finite difference expression representing the formulated governing differential equation for rectangular thin plate need to be formulated. Therefore to formulate the improved finite difference confidence for bending moment, the displacement of each node shown in Fig 3.1 is obtained using central difference method; hence considering the discretized plate shown in Fig (3.1), let f_0 *to* f_{48} represent the nodes on the discretized plate, therefore taking f_o as the pivoted point the required improved finite difference coefficient were obtained using equation 2.4. Thus Table 3.1 shows the derived improved finite difference coefficients for the second derivates acting along x and y axis.

Table 3.1: Summary of the High Order Improved Finite Difference Coefficients

Order of Derivatives	Order of Improved Finite Difference Coefficients
$\frac{dw}{dx}$	$\frac{1}{12h}(f_2 - 8f_3 + 8f_4 - f_5)$
$\frac{d^2w}{dx^2}$	$\frac{1}{12h^2}(-f_2 + 16f_3 - 30f_o + 16f_4 - f_5)$
$\frac{dw}{dy}$	$\frac{1}{12h}(f_{31} - 8f_{24} + 8f_{10} - f_{17})$
$\frac{d^2w}{dy^2}$	$\frac{1}{12h^2}(-f_{31} + 16f_{24} - 30f_o + 16f_{10} - f_{17})$

3.2 Formulating the Improved Finite Difference Expression for Bending Moment along x and y- axis by the Method of Higher Approximation:

Adopting the required improved finite difference derivative for the second order from Table 3.1 and substituting the expression into equation 2.1; thus carrying out further simplification will give the improved finite difference bending moment expression along x-axis. Hence the improved finite difference bending moment expression along x-axis is

$$-\frac{D}{12h^2}[(-30f_0 - 30v\alpha^2 f_0) + (16f_4) + (16f_3) + (-f_5) + (-f_6) + (-f_2) + (16v\alpha^2 f_{10}) + (16v\alpha^2 f_{24}) + (-v\alpha^2 f_{17}) + (-v\alpha^2 f_{31})] \quad (3.1)$$

Adopting the required improved finite difference derivative for the second order from Table 3.1 and substituting the expression into equation 2.2; thus carrying out further simplification will give the improved finite difference bending moment expression along y-axis. Hence the improved finite difference bending moment expression along y-axis is

$$-\frac{D}{12h^2}[(-30\alpha^2 f_0 - 30vf_0) + (16\alpha^2 f_{10}) + (16\alpha^2 f_{24}) + (-\alpha^2 f_{17}) + (-\alpha^2 f_{31}) + (16vf_4) + (16vf_3) + (-vf_5) + (-vf_6) + (-vf_2)] \quad (3.2)$$

Thus the mathematical model of the improved finite difference for bending moment along x and y-axis is shown in Figure 3.1 and 3.2

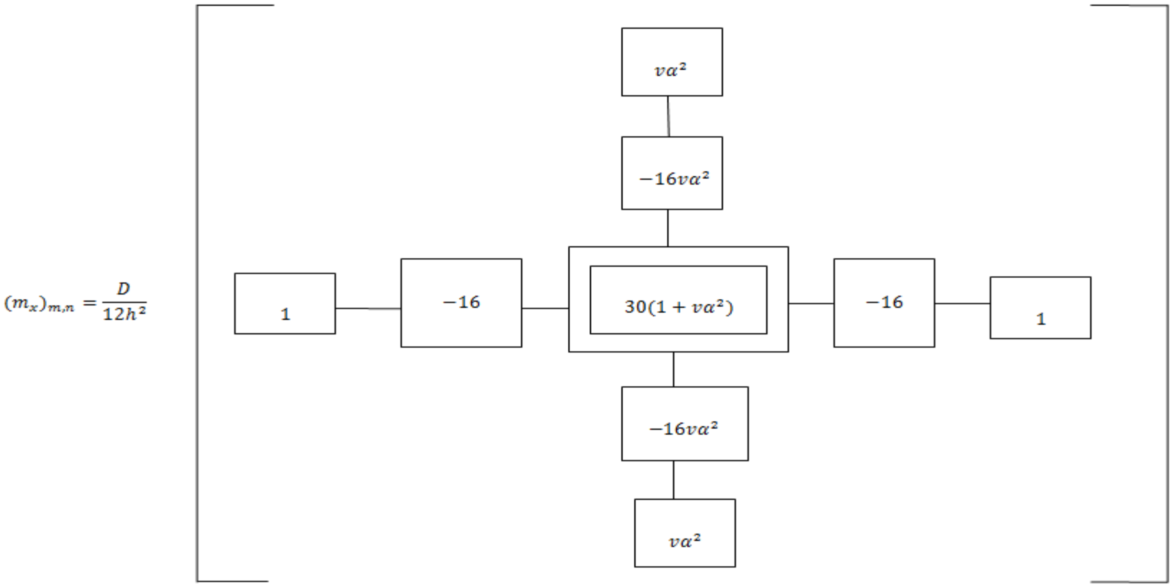


Figure 3.1 Mathematical Model of Improved Finite Difference for Bending Moment Analysis along X-axis

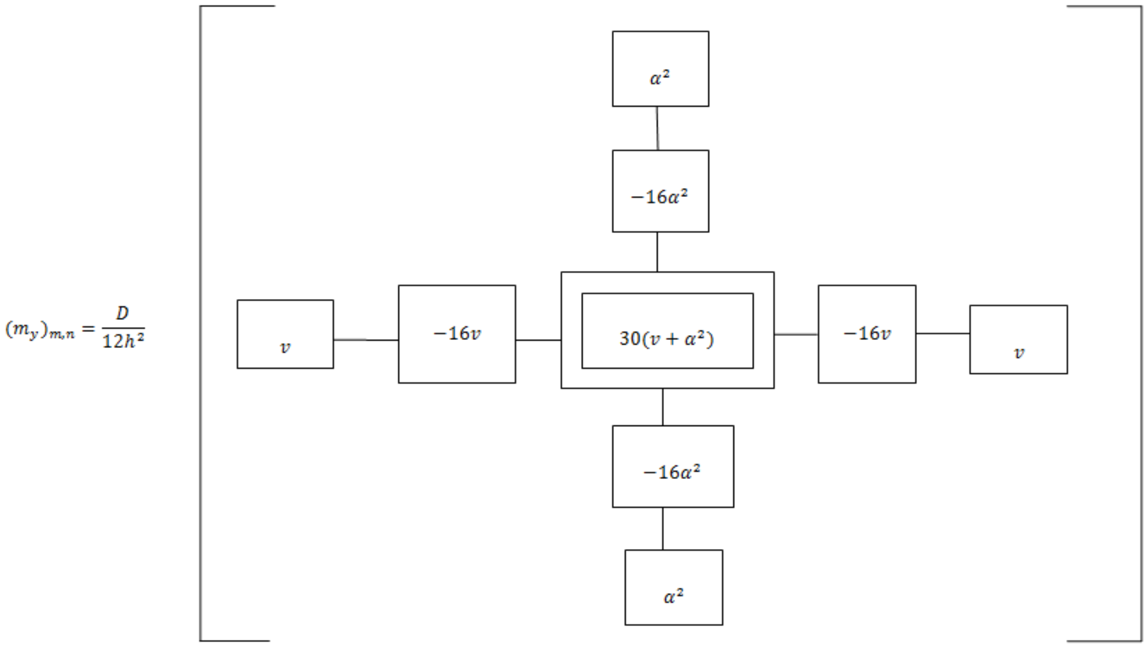


Figure 3.2 Mathematical Model of Improved Finite Difference for Bending Moment Analysis along y-axis

3.3 Boundary Conditions:

For a thin rectangular flat plate with all edge simply supported having an edge length $'a'$ and $'b'$ the boundary to be considered as zero are the shear force and bending moment (Szilard, 2004), thus considered the bending moment alone, the boundaries are

$$(m_x)_x = 0 \qquad (3.3)$$

$$(m_y)_y = 0 \qquad (3.4)$$

where

$$(m_x)_x = (\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2})_x \quad (3.5)$$

$$(m_y)_y = (\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2})_y \quad (3.6)$$

The improver finite difference forms are

$$(m_x)_x = \frac{1}{12h^2}(-f_2 + 16f_3 - 30f_o + 16f_4 - f_5) + v\frac{1}{12h^2}(-f_{31} + 16f_{24} - 30f_o + 16f_{10} - f_{17}) = 0 \quad (3.7)$$

$$(m_y)_y = \frac{1}{12h^2}(-f_{31} + 16f_{24} - 30f_o + 16f_{10} - f_{17}) + v\frac{1}{12h^2}(-f_2 + 16f_3 - 30f_o + 16f_4 - f_5) = 0 \quad (3.8)$$

Hence the boundaries can be expressed as

$$w_{x+1} = -w_{x-1} \quad (3.9)$$

4. NUMERICAL ANALYSIS

4.1 Generation of Nodes:

The interior nodes were generated using a step size of

$$\Delta a = \frac{a}{6} \; and \; \Delta b = \frac{b}{6} \qquad (4.1)$$

4.2 Application and Result of the Improve Finite Difference:

The bending moments were obtained by taking the inverse of the coefficient matrix and multiplying the inverse matrix with the right hand side of equation (4.2)

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix} = \frac{P(\Delta L)^4}{D} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (4.2)$$

Where $A_{11} \ldots . A_{33}$ is the coefficient matrix, $F1 \ldots F3$ is the deflection coefficient, P is the uniform load, (ΔL) is the step size and D is the depth of the plate.

4.3 Graphical Representation of Results:

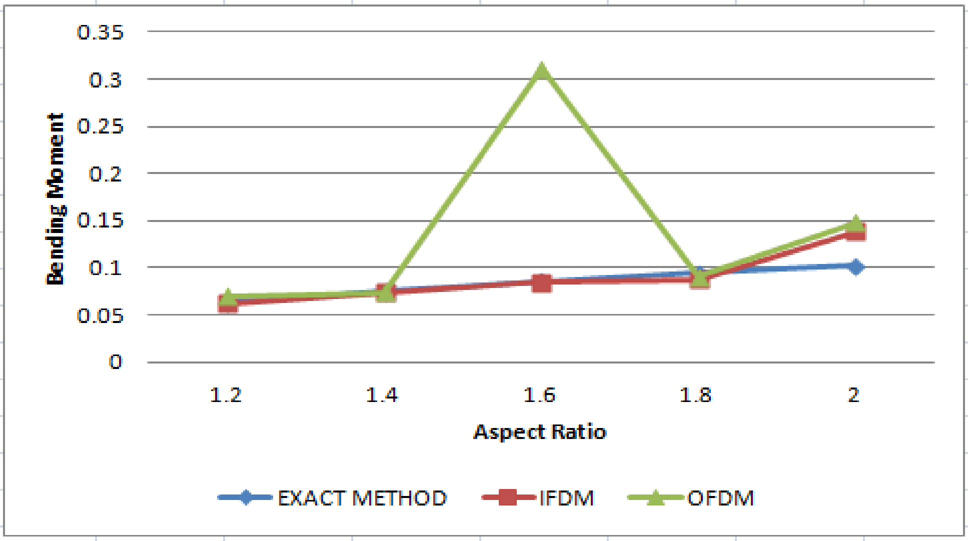


Figure 4.1: Comparison of the Numerical Solutions to the Exact Solution using Model for X-axis

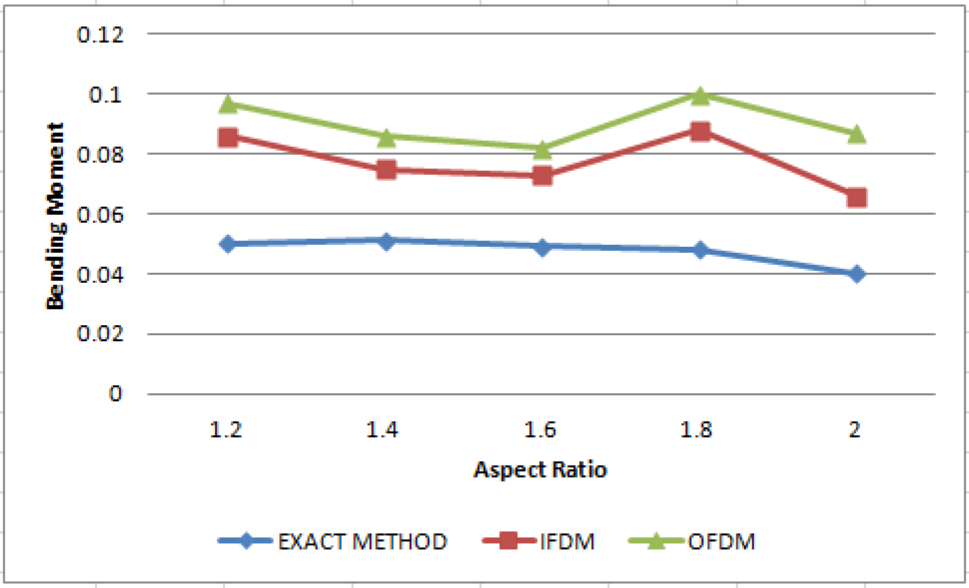


Figure 4.2: Comparison of the Numerical Solutions to the Exact Solution using Model for Y-axis

Graph plotted for maximum bending moment of various aspect ratios ranging from 1.2 to 2 for the numerical solution and exact solution as shown in figure 4.1 and 4.2 indicated that the results obtained using the improved finite difference method approximated closely to the exact solution than the ordinary finite difference method. Figure 4.3 and 4.4 shows the percentage errors between the numerical methods to the exact solution

Figure 4.3: Percentage Error of the Numerical Solutions to the Exact Solution Using Model of X-axis

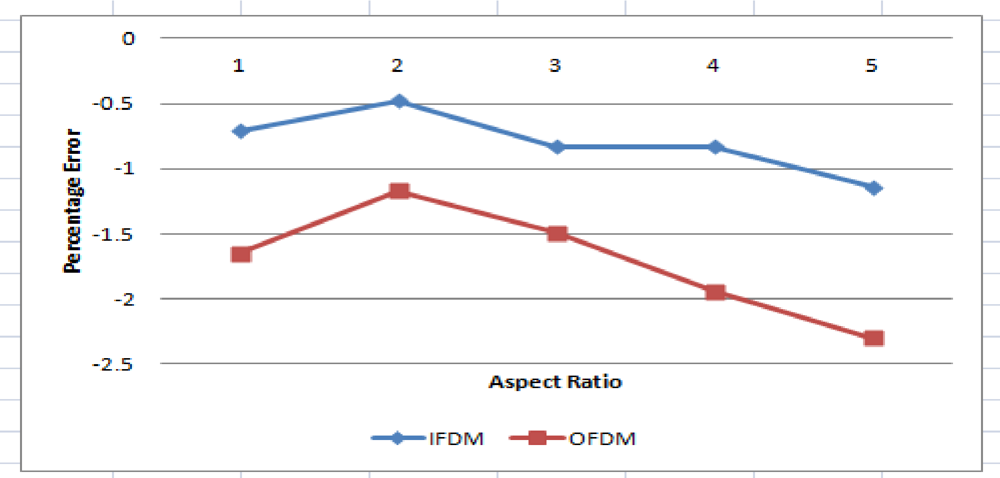


Figure 4.4: Percentage Error of the Numerical Solutions to the Exact Solution Using Model for Y-axis

Graph plotted for percentage error of the numerical solutions to the exact solution as shown in figure 4.3 and 4.4 using the X and Y axis models indicated that the improved finite difference results using the X-axis model showed an average percentage error of -0.048% to the exact solution while the ordinary finite difference solution showed a percentage error of -3.035% to the exact solution.

The improved finite difference results using the Y-axis model shows a percentage error of -0.798% to the exact solution while the ordinary finite difference showed -0.912% to the exact solution.

5. CONCLUSIONS

The following conclusions were made

i. The analysis indicates that the maximum bending moment of the plate will occurred at the centre.

ii. The converged to the exact solution was close and rapid.

iii. The use of improved finite difference by the method of higher approximation proved to be more accurate than the ordinary finite difference method.

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